MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/2022

PCL0016 – CALCULUS

(ALL Sections/Groups)

3 OCTOBER 2022
2:30 P.M - 4:30 P.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 4 pages including cover page with FOUR questions.

2. Attempt **ALL FOUR** questions and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1 (25 marks)

(a) Evaluate the following limits without using L'Hôpital's Rule.

(i) $\lim_{x\to -2} \dfrac{x+3}{x-2}$ [2 marks]

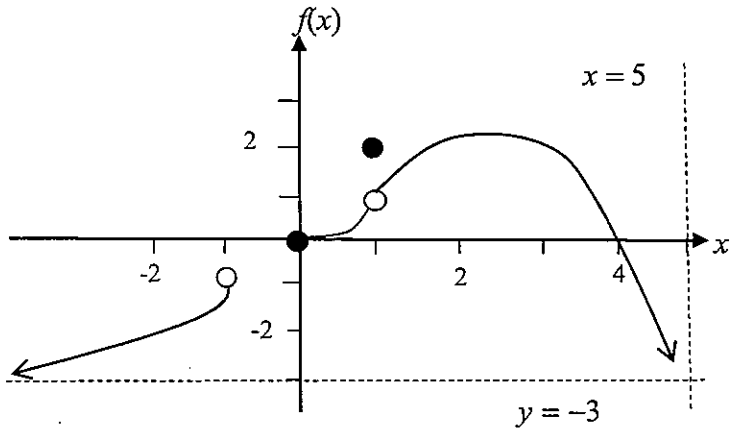
(ii) $\lim_{x\to 7} \dfrac{x^2+4x-77}{x^2-9x+14}$. [3 marks]

(iii) $\lim_{x\to \infty} \dfrac{x^5-x^3+x}{x^3+2x^2+1}$. [3 marks]

(iv) $\lim_{x\to 0} \dfrac{\sin^5 x}{\tan^4 x}$. [3 marks]

(v) $\lim_{x\to 3} \dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}$ [3 marks]

(b) The graph of a function $f(x)$ is given below.



Find the following limits:

(i) $\lim_{x\to 1^-} f(x)$ [1 mark]

(ii) $\lim_{x\to 4} f(x)$ [1 mark]

(iii) $\lim_{x\to 5} f(x)$ [1 mark]

Continued.....

(iv) $\lim_{x\to-\infty} f(x)$ [1 mark]

(v) $f(1)+\lim_{x\to0^+} f(x)$ [2 marks]

(c) Is the function $f(x)$ in part (b) continuous at $x=0$ and at $x=1$? Justify your answers by using the conditions for continuity at a point. [5 marks]

Question 2 (25 marks)

(a) Evaluate $\frac{dy}{dx}$ for the following functions:

(i) $y=-\frac{x^2}{16}+\frac{2}{x}-x^{3/2}+\frac{1}{\sqrt[4]{x}}$. [2 marks]

(ii) $y=\ln\left(e^{2x}+3\right)$ at $x=0$. [3 marks]

(iii) $y=\frac{x^2e^{3x}}{\sqrt{x-1}}$ by using logarithmic differentiation. [6 marks]

(b) Evaluate the following integrals:

(i) $\int\frac{4\ln x}{x^2}dx$ by using integration by parts. [4 marks]

(ii) $\int_0^1(2x)e^{x^2+1}dx$ by using integration by substitution. [4 marks]

(iii) $\int\frac{x+1}{(x-5)(x-8)}dx$ by using partial fraction decomposition. [6 marks]

Continued.....

Question 3 (25 marks)

(a) A region is bounded by $y = x^3$, $y = 27$ and $x = 0$. Sketch the bounded region and find the following:

(i) Area of the bounded region. [5 marks]

(ii) Volume of the solid generated by revolving the bounded region about the y-axis by using disk method. Then, verify the answer by using the cylindrical shell method. [9 marks]

(b) A force of 30N is required to maintain a spring stretched from its natural length of 12cm to a length of 15cm. How much work is done in stretching the spring from 12cm to 20cm? [5 marks]

(c) Find the average value of $f(x) = 2\sin x - \sin 2x$ on the interval $[0, \pi]$. [6 marks]

Question 4 (25 marks)

(a) Solve the following differential equations.

(i) $\dfrac{dy}{dx} = \dfrac{\sin x}{y}$. [5 marks]

(ii) $\dfrac{dy}{dx} + \dfrac{3}{x}y = \dfrac{1}{x^2} - \dfrac{1}{x^3}$. [6 marks]

(b) Find the particular solutions of the following differential equations subject to the given initial values.

(i) $2x\dfrac{dy}{dx} = 2y\ln x, \quad y(1) = 2$. [7 marks]

(ii) $16y'' - 8y' + y = 0, \quad y(0) = \dfrac{1}{2}, y'(0) = -2$ [7 marks]

End of Paper